HS Target F

Domain, Target, Standards, DOK, Vertical Alignments, Achievement Levels, Evidence Required, Vocabulary, Response Types, Materials, Attributes, Question Types, and Question Banks (Examples)

Content Domain: Algebra
Target F [m]: N-RN.A Extend the properties of exponents to rational exponents.
Standards included in Target F: A-APR.A Perform arithmetic operations on polynomials.
Vertical Alignment
Achievement Level Descriptors
Evidence Required
Vocabulary
Response Types
Materials
Attributes
Claim 1: Concepts and Procedures (DOK 2) Question Banks
Claim 2 Problem Solving Questions Banks
Claim 3 Communicating Reasoning Question Banks

Content Domain: Algebra

Target F [m]: N-RN.A Extend the properties of exponents to rational exponents.

Standards included in Target F: A-APR.A Perform arithmetic operations on polynomials.

A-APR.A Perform arithmetic operations on polynomials.

A-APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Vertical Alignment

Related Grade 8 standards

8.EE.C Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.C.7 Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers).

b. Solve linear equations with rational number coefficients, including equations whose

solutions require expanding expressions using the distributive property and collecting like terms.

Achievement Level Descriptors

Level 1 Students should be able to add, subtract, and multiply single variable polynomials of degree 2 or less.

Level 2 Students should be able to add, subtract, and multiply multivariable polynomials made up of monomials of degree 2 or less. They should understand that polynomials are closed under addition.

Level 3 Students should be able to add, subtract, and multiply multivariable polynomials of any degree and understand that polynomials are closed under subtraction and multiplication.

Level 4 Students should understand and be able to explain that polynomials form a system analogous to the integers.

Evidence Required

1. The student adds or subtracts polynomials.

2. The student multiplies polynomials.

Vocabulary
terms, factors, coefficients, monomials, binomials, trinomials, polynomials, exponents, expressions, distribute, distributive property, sum, difference, product, like terms

Response Types
Equation/Numeric; Multiple Choice, single correct response

Materials
Two or more polynomials

Attributes
None

Claim 1: Concepts and Procedures (DOK 2) Question Banks
Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Claim 1 A-APR.A.1 DOK Level 2

Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

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Evidence Required
The student adds or subtracts polynomials.

Question Type 1: The student is presented with a polynomial expression and is required to add and/or subtract polynomials in order to write it in another form.

1. Enter an expression that is equivalent to $(4x^2-5x+6)+(9x^2-2x)-(11x-3)$, combining all like terms.

Rubric: (1 point) The student enters a correct expression (e.g., $13x^2-18x+9$).

Response Type: Equation/Numeric

Question Type 2: The student is presented with a polynomial expression and is required to identify the expression written in another form.

1. Which expression is equivalent to $(mx+5)+(2x-b)$?

A. $2mx-5b$
B. $(2+m)x-b+5$
C. $2mx-5+b$
D. $2mx-bmx+10x-5b$

Rubric: (1 point) The student selects the correct expression (B).

Response Type: Multiple choice, single correct response

Claim 1 A-APR.A.1 DOK Level 2

Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Evidence Required:
The student multiplies polynomials.

Question Type 1: The student is presented with an expression involving the product of polynomials and directed to respond in a specific form.

1. Enter an expression equivalent to $(-\frac{1}{2}at)\cdot(12t^3)$ in the form A $x^m y^n$.

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Rubric: (1 point) The student enters the product in the requested form (e.g.,$-6at^4$).

Response Type: Equation/Numeric

Question Type 2: The student is presented with an expression involving the product of polynomials.

1. Multiply and combine like terms to determine the product of these polynomials.

$(2n-3)(5n+6)$

Enter your result in the response box.

Rubric: (1 point) The student correctly multiplies and combines like terms (e.g., $10n^2-3n-18$).

Response Type: Equation/Numeric

Question Type 3: The student is presented with an expression involving the product of polynomials.

1. Which expression is equivalent to $(2x-t)\cdot(3x+5)$?

   A. $5x-t+5$
   B. $6x^2-5t$
   C. $6x^2+7x-5t$
   D. $6x^2+(10-3t)x-5t$

2. Which expression is equivalent to $(aaxx+bb)\cdot(ccxx+dd)$?

A. $acx2+bd$
B. $(a+c)x+(b+d)$
C. $(a+c)x^2+(b+d)$
D. $acx^2+(ad+bc)x+bd$

Rubric: (1 point) The student selects the correct expression (e.g., C; D).

Response Type: Multiple choice, single correct response

Claim 2 Problem Solving Questions Banks

Claim Descriptors and Targets

Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.

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Example 1

Find all the zeros of the following polynomial function and enter them into the boxes.

$f(x)=x^5-37x^3-24x^2+180x$

Zeros:

Interaction: The student has access to a graphing calculator which can be used to identify the zeros of the polynomial.

Rubric: (1 point). Student enters the correct zeros in the response boxes (–6, –2, 0, 3, 5; the numbers may be in any order).

Response type: Fill-in Table

Claim 3 Communicating Reasoning Question Banks

Claim Descriptors and Targets

Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

Example 1

Kiera claimed that the sum of two linear polynomials with rational coefficients is always a linear polynomial with rational coefficients.

Drag the six statements into a logical sequence to outline an argument that proves this claim.

1.	$p(x) + q(x) = (ax+cx) + (b+d)$
2.	$p(x) + q(x) = (ax+b) + (cx+d)$
3.	Given $p(x) = ax+b$ and $q(x) = cx+d$ where a, b, c, and d are rational numbers.
4.	$(a+c)$ and $(b+d)$ are rational numbers
5.	So $p(x) + q(x)$ is a linear polynomial with rational coefficients.
6.	$p(x) + q(x) = (a+c)x + (b+d)$

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Rubric: (2 points) The student completes and orders the four statements that support the claim in a logical order (Example below. Note that 1 must come first, 6 must come last, the order of 2, 3, and 4 must be preserved but 5 can go anywhere in between 1 and 6).
(1 point) The student completes and orders the four statements that support the claim in a logical order but either chooses some of the options from the drop-down menus incorrectly or chooses them in a sensible way if the order were correct, but the order is not correct.
Example

1. Given p(x) = ax+b and q(x) = cx+d where a, b, c, and d are rational numbers.
2. p(x) + q(x) = (ax+b) + (cx+d)
3. p(x) + q(x) = (ax+cx) + (b+d)
4. p(x) + q(x) = (a+c)x + (b+d)
5. (a+c) and (b+d) are rational numbers
6. So p(x) + q(x) is a linear polynomial with rational coefficients.

Response Type: Drag and Drop

Example 2

Proposition 1 The sum of two linear polynomials with integer coefficients is always a linear polynomial with integer coefficients.

The Closure of the Integers Under Addition The sum of two integers is always an integer

The Commutative Property If A and B are real numbers, then A + B = B + A

The Associative Property If A, B, and C are real numbers, then (A + B) + C = A + (A + C)

The Distributive Property If A, B, and C are real numbers, then A(B + C) = AB + AC

The Any-Order Property of Addition The sum of two or more real numbers can be performed in any order or any grouping.

The outline of a proof of Proposition 1 is shown. Add one or more justifications for steps 3, 4, and 5 of the proof.

Statement	Justification
1. Given $p(x) = ax+b$ and $q(x) = cx+d$ where a, b, c, and d are integers.	Hypothesis
2. $p(x) + q(x) = (ax+b) + (cx+d)$	By Definition
3. $p(x) + q(x) = (ax+cx) + (b+d)$	
4. $p(x) + q(x) = (a+c)x + (b+d)$	
5. $(a+c)$ and $(b+d)$ are integers	
6. So $p(x) + q(x)$ is a linear polynomial with integer coefficients.	Conclusion

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Rubric: (1 point) The student provides a correct justification for each step of the proof (Example below. Note that for step (3), students can either cite the any-order property of addition or cite the commutative and associative properties together).

Statement	Justification
1. Given $p(x) = ax+b$ and $q(x) = cx+d$ where a, b, c, and d are integers.	Hypothesis
2. $p(x) + q(x) = (ax+b) + (cx+d)$	By Definition
3. $p(x) + q(x) = (ax+cx) + (b+d)$	**The Any-Order Property of Addition**
4. $p(x) + q(x) = (a+c)x + (b+d)$	**The Distributive Property**
5. $(a+c)$ and $(b+d)$ are integers	**The Closure of the Integers Under Addition**
6. So $p(x) + q(x)$ is a linear polynomial with rational coefficients.	Conclusion

Response Type: Drag and Drop

Example 3

P(x) is a 4th degree polynomial. The graph of y = P(x) has exactly three distinct x-intercepts.

Which polynomial could be P(x)?

A. $x^3(x-3)$
B. $x^2(x-2)(x-1)$
C. (x-3)(x-2)(x-1)
D. x(x-3)(x-2)(x-1)

For one of the polynomials above, explain why it could not be P(x).
The graph of y = [drop-down choices: the four polynomials listed in the table] has exactly [drop-down choices: 0, 1, 2, 3, 4] distinct x-intercepts at x = [0] [1] [2] [3] [4]

Click on all of the distinct x-intercepts

The degree of this polynomial is [drop-down choices: 0, 1, 2, 3, 4].

Rubric: (2 points) The student clicks on the correct polynomial (B) and selects one of the polynomials, identifies the correct number of distinct x-intercepts, correctly identifies the x-intercepts, and correctly identifies the degree of the polynomial. (See Example below. This is just one possibility.)
(1 point) The student clicks on the polynomials that could be P(x) or fills out the information about one of the polynomials correctly.

The graph of y = x3(x-3) has exactly 2 distinct x-intercepts at

- x = [0] [1] [2] [3] [4]

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The degree of this polynomial is 4.

Response Type: Multiple Choice, single correct response and Drop Down Menu

Example 4

An algebraic identity is an equation that is always true for any value of the variables.

For example, 2(x+y) = 2x+2y is an identity because of the distributive property.

For each equation, select “Yes” if the equation is an algebraic identity. Select “No” if it is not an algebraic identity.

Equation	Yes	No
$(x+3)^2 = x^2 + 3^2$		
$y^{-1} = \frac{1}{y}$ if $y \neq 0$		
$a^2 + b^2 = c^2$		

Rubric: (1 point) The student correctly indicates which equation is an identity and which is not (NYN).

Response Type: Matching Table